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Theoretical and experimental investigation of the effects of an irregular-pitch cutter on vibration in face-milling †

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Abstract

In the present work, cutting-force models of single-tooth and multi-teeth face-milling cutters were set up. Based on a spectrum analysis of cutting force, the vibration mechanism of a face-milling cutter of irregular pitch was investigated theoretically. The single-objective function and constraint conditions were derived. A general-purpose irregular-pitch face-milling cutter subsequently was designed and tested, and its performance was compared with that of a regular-pitch cutter. The experimental results showed that the irregular-pitch face-milling cutter not only reduces vibration and noise but also enhances surface-finish quality.

Keywords: Cutting force; Face-milling cutter; Vibration-free; Optimum design

1. Introduction

Metal-cutting processes, especially at high metal removal rates, often are disturbed by an intense vibration force. This vibration can considerably shorten the life of a cutter. For longer machine tool life, therefore, it is necessary to improve the dynamic stability of milling operations. Dynamic stability can be enhanced by improving the dynamic behavior of the machine-tool-fixture-workpiece (MTFW) system. Another possible solution is a cutter of irregular pitch. Much work in fact has already been devoted to such a design. Slavicek [1] unveiled a chatter-suppressing version of this cutter type, which, however, can be used only under special cutting conditions. H. Optiz [2] and P. Vanherck [3] designed a cutter based on a chatter-suppression stability theory. P. Doolan et al. [4-6] designed a special-purpose cutter that can reduce vibration substantially under a certain range of cutting conditions, as based on an impulse or rectangular-pulse cutting-force model. In the present work, a cutting-force calculation model was first derived, on the basis of which a model for an irregular-pitch face-milling cutter was established. The vibration mechanism was investigated by analyzing the spectrum of the milling forces. Subsequently, the model was optimized based on cutting-force spectral redistribution criteria. Finally, a general-purpose irregular-pitch face-milling cutter with uneven blade spacing was designed and its performance compared with that of a regular-pitch cutter.

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2. Determination of cutting-force components in face milling

Fig. 1 shows the cutting-force components as a function of both the cutter and cutting geometries. From the cutting geometry shown in Fig. 1, the diameter of the cutter, D, the width of the workpiece AE, the distance from the centre of the workpiece to the centre of the cutter, CD, the feed speed, V_f , the unformed width of the cut, a_w , the chip thickness, a_c , the contact angle, ϕ_s , and the ith insert at the cutter rotation angle ϕ , are given as

$$\phi_s = \phi_e - \phi_0 \tag{1}$$

$$a_w = a_p / \sin \chi_r \tag{2}$$

$$a_c = a_f \times \sin\phi \times \sin\chi_r \tag{3}$$

where ϕ_0 is the entry angle, ϕ_e is the exit angle, a_p is depth of cut, χ_r is the approach angle of the cutter and a_f is the feed per tooth.

In face-milling operations it is convenient to consider the tangential force components, the most important of which, the power force p_t , can be expressed as [10]

$$p_t = C \times a_w \times a_c^{\lambda} . \tag{4}$$

Substituting Eq. (2) and Eq. (3) into Eq. (4), one obtains

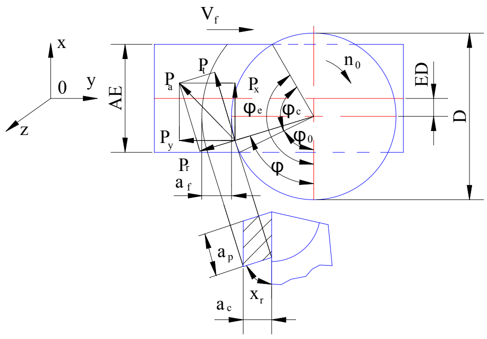

Fig. 1. Cutting-force components in face milling.

$$p_t = C \times (a_p / \sin \chi_r) \times (a_f \times \sin \chi_r \times \sin \varphi)^{\lambda} \tag{5}$$

where C, λ are the coefficient and index number, which are determined experimentally.

Many authors have proposed that the radial force p_r acting along the cutting edge in the radial direction of the cutter is obtained by multiplying the tangential force by an empirical constant, η [12]. Neglecting the effects of tool geometry (the rake and lead angles), the radial force and axial force are given as

$$p_r = \eta \times p_t \times \sin \chi_r \tag{6}$$
$$p_z = \eta \times p_t \times \cos \chi_r \tag{7}$$

where η is the empirical constant for cutting condition, η =0.1~0.3.

The milling force components acting on the ith insert in the X-Y-Z coordinate system are given as [10]

$$\begin{aligned}\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} &= \begin{bmatrix} -\cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & -\cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} P_r \\ P_t \\ P_z \end{bmatrix} \\ &= \begin{bmatrix} -\eta \times \sin\chi_r \times \cos\varphi & \sin\varphi & 0 \\ -\eta \times \sin\chi_r \times \sin\varphi & -\cos\varphi & 0 \\ 0 & 0 & \eta \times \cos\chi_r \end{bmatrix} \times P_t \end{aligned} \tag{8}$$

Fig. 2 shows the principle underpinning irregular-pitch face-milling. Let $\theta_0, \theta_1, ..., \theta_{z-1}$ be the blade angle in a cutter having Z number of blades. θ_0 may be arbitrarily taken to be zero. The angle of cutter rotation, which is a function of time t, can be given as

$$\beta = 2\pi n_0 \times t / 60 \tag{9}$$

where n_0 is the cutter speed in rpm and t is the time in seconds.

The instantaneous milling force can be obtained as the resultant of all forces acting on the individual inserts engaged in cutting at a given instant. Based on this conventional milling model assumption, the multi-tooth milling force on the work-

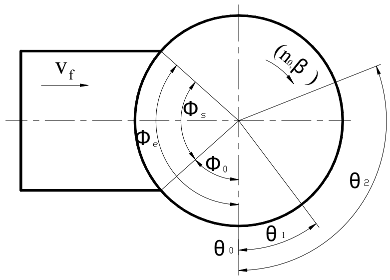

Fig. 2. Cutting principle of irregular-pitch cutter.

piece can be characterized as [10]

$$p_t(i,\beta) = \begin{cases} C \times (a_p / \sin\chi_r) \times [q_i \times a_{fav} \times \sin\chi_r \times \sin\varphi_i(\beta)]^{\lambda} \\ \qquad (2\pi m + \varphi_0 + \theta_i) \le \beta \le (2\pi m + \varphi_e + \theta_i) \\ \qquad\qquad m = 0, \pm1, \pm2, \pm3, \ldots\ldots \\ 0 \qquad\qquad , \quad otherwise \end{cases} \tag{10}$$

where $q_i = Z \times (\theta_i - \theta_{i-1}) / 2\pi$, $q_i \times a_{fav}$ is the amount of feed of the ith tooth, θ_i is the angular position of the blade expressed in radians, and $\varphi_i(\beta)$ is the instantaneous feed direction angle, which can be expressed as

$$\varphi_i(\beta) = \beta - 2\pi m - \theta_i . \tag{11}$$

In the X-Y-Z coordinate system, the milling force components can be given as [10]

$$\begin{bmatrix} P_x(\beta) \\ P_y(\beta) \\ P_z(\beta) \end{bmatrix} = \sum_{i=0}^{z-1} \begin{bmatrix} -\eta \times \sin\chi_r \times \cos\varphi_i(\beta) + \sin\varphi_i(\beta) \\ -\eta \times \sin\chi_r \times \sin\varphi_i(\beta) - \cos\varphi_i(\beta) \\ \eta \times \cos\chi_r \end{bmatrix} \times p_t(i,\beta) . \tag{12}$$

3. Spectrum analysis of cutting force

The cutting force of a single tooth can be regarded as a $\delta - function$. Accordingly,

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \ne 0 \end{cases} \text{ and } \int_{-\infty}^{\infty} \delta(t) dt = 1 . \tag{13}$$

Let $p_0(t) = [p_x(t) \quad p_y(t) \quad p_z(t)]^T$ be the cutting force of a single tooth acting in the X, Y, Z directions, respectively. The cutting force of an irregular-pitch face-milling cutter, then, can be expressed as

$$p_z(t) = p_0(t) * \delta_T(t) \tag{14}$$

where $\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - n \times t_z)$, $t_z = 60/(n_0 \times Z)$, n_0 is the cutter speed in rpm and Z is the number of blades, n=1,2,3....

According to time-domain convolution theory, the Fourier

Series Transform of Eq. (14) is completed. Therefore, the frequency spectrum of milling forces for an irregular-pitch face-milling cutter is

$$p_z(f) = f_z \times p_0(f = n \times f_z) \times \sum_{n=-\infty}^{\infty} \delta(f - n \times f_z) \tag{15}$$

where f_Z is the impact frequency of the insert and $f_Z = 1/t_z$. Eq. (15) shows that the force spectrum of a face-milling cutter of regular pitch is only presented to integral times of frequency $f(f = n \times f_Z)$. Therefore, the relative vibration is more concentrated.

However, when cutting using a cutter of irregular pitch, the neighboring blade spacing θ_i is unequal. So, based on the $\delta - function$, the cutting force of an irregular-pitch cutter is given by

$$p_B(t) = p_0(t) * G_f(t) * \delta_T(t) \tag{16}$$

where $G_f(t)$ is the distribution function of blade spacing, which can be represented as

$$G_f(t) = \sum_{i=1}^{z} q_i^{\lambda} \times \delta(t - \sum_{k=1}^{i-1} \tau_k) \quad (\text{k=1,2,3,……}) \tag{17}$$

where τ_k is the rotation time for single blade spacing with a cutter of irregular pitch.

According to time-domain convolution theory and the Fourier Series Transform, the frequency spectrum of the cutting force of an irregular-pitch face-milling cutter is given by

$$p_B(f) = f_n \times G_f(f_k) \times p_0(f_k) \qquad (k = 1,2,3,......) \tag{18}$$

where $f_n = n_0 / 60$ is the cutter rotation frequency and $f_k = k \times f_n$ is the sampling frequency needed to obtain the discrete spectrum.

Eq. (18) shows that the cutting-force spectrum of a face-milling cutter of irregular pitch is mainly presented to the spectrum of a single blade. But the distribution function of blade spacing $G_f(f)$ has a more significant effect on that spectrum. The amplitude spectrum is dispersed at a special frequency by which the spacing is equal to the cutter rotation frequency f_n. Because the excitation power is distributed much more frequently, the vibration power is more decentralized, and each frequency amplitude is smaller.

4. Vibration mechanism of irregular-pitch face-milling cutter

If the dynamic system is known, the relative vibration between the face-milling cutter and the workpiece can be written as [4]

$$S_R(f) = A_e(f) \times S_s(f) \tag{19}$$

where $A_e(f)$ is the amplitude spectrum of the excitation force and $S_s(f)$ is the amplitude spectrum of the MTFW.

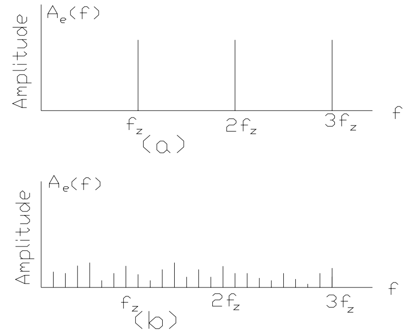

Fig. 3. (a) Amplitude spectrum of excitation force for cutter with equal blade spacing. (b) Amplitude spectrum of excitation force for cutter with unequal blade spacing.

Eq. (15) shows that the cutting-force spectrum of an irregular-pitch cutter only presents to integral multiple of frequency $f(f = n \times f_Z)$. Contrastingly, the amplitude spectrum of an irregular-pitch cutter presents a more decentralized frequency. The relation of the two frequencies is given as

$$f_Z = Z \times f_n . \tag{20}$$

A conclusion can be drawn: Since the force spectrum of an irregular-pitch cutter is distributed over a wider distributed discrete frequency band, the force amplitude becomes not only much smaller but also flatter. Consequently, the vibration force of the cutting process is reduced and controlled. In other words, the irregular pitch breaks up the pure-tone nature of the impacts that would be caused by a regular pitch. Thereby, the dynamic stability of the MTFW system is enhanced. Computer simulation results are shown in Fig. 3. Fig. 3(a) shows the amplitude spectrum of the excitation force for a regular-pitch cutter, and Fig. 3(b), that for an irregular-pitch cutter.

5. Determination of blade spacing for general-purpose cutter

When the dynamic frequency response of an MTFW system is known, a special-purpose cutter can be designed to minimize the relative vibration for a particular cutting speed. When the dynamic system response is unknown, the design choice is limited to a general-purpose cutter so as to avoid resonant excitation over a broad frequency range.

In the present work, a general-purpose face-milling cutter was designed. When a face- milling cutter is to be used for a large number of different operations, at different speeds on many different machines, the frequency response function of the MTFW system will not be available. And when this function is unknown, it is necessary to assume a characteristic response to be used with the design method proposed. It may be assumed that [4]

$$S_s(f) = cons\tan t, \qquad for\,all \quad f\ . \tag{21}$$

The Fourier series representation of $p_j(t)$ can be written as

$$P_j(t) = a_0 + \sum_{n=1}^{\infty}[a_n \times \cos(n\omega_n t) + b_n \times \sin(n\omega_n t)] \tag{22}$$

where

$$a_0 = \frac{1}{2\pi}\int_0^{\frac{2\pi}{\omega_n}} P_j(t)dt$$
$$a_n = \frac{1}{\pi}\int_0^{\frac{2\pi}{\omega_n}} P_j(t) \times \sin(n\omega_n t)dt$$
$$b_n = \frac{1}{\pi}\int_0^{\frac{2\pi}{\omega_n}} P_j(t) \times \cos(n\omega_n t)dt$$

and where ω_n is the circular frequency of the cutter and j represents the X, Y, or Z coordinate direction. Then, the amplitude of the force function $p_j(t)$ at the nth harmonic is given by

$$A_n = \sqrt{a_n^2 + b_n^2}\ . \tag{23}$$

It is possible that a machine tool has different stiffnesses in different directions X, Y, and Z. In order to resolve this issue, normalized weight coefficients are given as

$$p_x + p_y + p_z = 1 \tag{24}$$

where p_x, p_y, p_z are the weight coefficients in the X, Y, Z directions, respectively, as determined by the stiffness of the machine tool. Hence, we can transform a multi-objective function into a single-objective function. The objective function, then, may be expressed as

$$A(\theta) = Min[p_x \times MaxA_{xk} + p_y \times MaxA_{yk} + p_z \times MaxA_{zk}] \tag{25}$$

where $MaxA_{xk}$ is the maximum of the force amplitude in the X direction, $MaxA_{yk}$ is the maximum of the force amplitude in the Y direction, and $MaxA_{zk}$ is the maximum of the force amplitude in the Z direction.

The objective function chooses blade spacing $(\theta_0, \theta_1, \theta_2, ...\theta_{z-1})$ such that the Fourier amplitude spectrum is as flat as possible. The linear objective function $A(\theta)$ is minimized by application of the constrained stochastic direction method, which enables high convergence speed and precision. The blade angle is constrained by the structure of the cutter, the strength of the blade and the wear of the inserts. Hence, the objective function is subjected to the practical and geometric design-variable constraints:

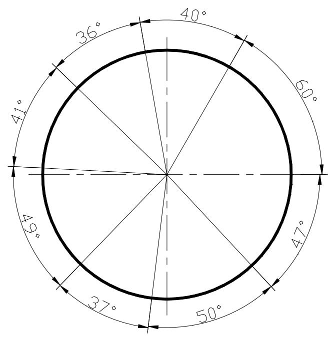

Fig. 4. Blade spacing of cutter.

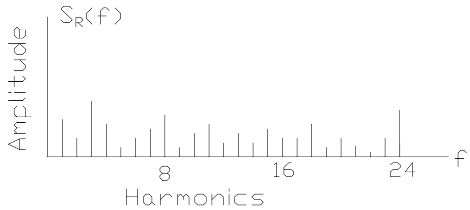

Fig. 5. Amplitude spectrum of relative vibration.

$$\begin{cases} \theta_{\min} \le \theta_i \\ \sum_{i=1}^{z} = 2\pi \\ \theta_{i\max} - \theta_{i\min} \le \Delta \end{cases} \tag{26}$$

where $\theta_{i\max}$ is the maximum blade angle, $\theta_{i\min}$ is the minimum blade angle, and Δ is the difference between them, as determined by a designer in accordance with the structure of the cutter.

In the present study, the above theory was applied to the design of a new face-milling cutter. Consider a general-purpose cutter of ϕ 160 mm diameter and eight blades. Suppose that the difference between the maximum and minimum blade angles Δ is 15° and the minimum allowed blade angle $\theta_{\min}$ is 35°. The optimum blade spacing and corresponding amplitude spectrum are shown in Figs. 4 and 5, respectively. Note that compared with the excitation spectrum of a regular-pitch face-milling cutter, the amplitude spectrum of an irregular-pitch cutter is much flatter.

6. Experimental procedure and results

The aim of the experiments was to compare the effectiveness of the designed irregular-pitch face-milling cutter with that of a similar-geometry regular-pitch cutter in reducing vibration force and noise and in improving the quality of the surface roughness of a workpiece. The regular-pitch cutter available for study is of 160 mm diameter, has eight regular blades, and uses carbide tips as inserts (SANDVICK model R-262.2-160-14). The experiments were run on a knee-and-column vertical milling machine (model X5030A). The relative vibration, the acoustic noise and the surface roughness of the workpiece

Table 1. Experimental results for vibration acceleration.

Cutting conditions	Type of cutter	Maximum vibration acceleration (mm/s^2)		
		X	Y	Z
n_0=220 r/min V_f=160 mm/min a_p=3 mm	Equal blade cutter	7.0	24.2	21.5
	Unequal blade cutter	5.4	14.8	16.3
n_0=220 r/min V_f=120 mm/min a_p=3 mm	Equal blade cutter	6.0	21.7	18.3
	Unequal blade cutter	4.6	12.9	13.2
n_0=270 r/min V_f=190 mm/min a_p=3 mm	Equal blade cutter	7.3	22.5	19.7
	Unequal blade cutter	5.2	14.7	13.9
n_0=270 r/min V_f=120 mm/min a_p=3 mm	Equal blade cutter	6.0	23.4	17.6
	Unequal blade cutter	4.4	13.8	12.1
n_0=385r/min V_f=270mm/min a_p=3mm	Equal blade cutter	10.2	31.3	22.3
	Unequal blade cutter	6.4	18.0	19.7
n_0=385 r/min V_f=190 mm/min a_p=3 mm	Equal blade cutter	8.6	25.7	19.2
	Unequal blade cutter	5.6	12.9	13.3
n_0=385 r/min V_f=120 mm/min a_p=3 mm	Equal blade cutter	6.3	19.5	15.9
	Unequal blade cutter	5.0	10.7	11.7

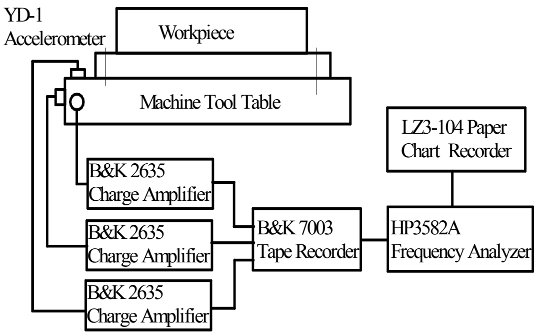

Fig. 6. Vibration measurement set-up.

were investigated.

The relative vibration of the workpiece caused by the cutting operation was measured for both cutters. The workpiece material was #45 steel of HB187 hardness, a 300 mm × 160 mm × 100 mm block of which was cut on the milling machine. A schematic of the vibration test apparatus is shown in Fig. 6. An accelerometer (model YD-1), a charge amplifier (model B&K 2635), a paper chart recorder (model LZ3-104), a tape recorder (B&K 7003) and a frequency analyzer (model HP3582A) were utilized to determine the relative vibration. The amplitudes of the vibration incurred in machining with the irregular-pitch and regular-pitch cutters were recorded. The maximum vibration accelerations of the cutters are listed in Table 1.

The acoustic noise also was measured, using the noise test apparatus schematized in Fig. 7. A B&K condenser microphone, sound level analyzer, and octave-band filter were used to measure the overall noise level. The background noise level was 69dBA. Table 2 lists the experimental results, which

Table 2. Experimental results for acoustic noise level.

Cutting condition	Type of cutter	Acoustic noise level (dBA)
n_0=385 r/min V_f=190 mm/min a_p=3 mm	Equal blade cutter	87
	Unequal blade cutter	84
n_0=270 r/min V_f=190 mm/min a_p=3 mm	Equal blade cutter	86
	Unequal blade cutter	83

Table 3. Experimental results for surface roughness.

Cutting condition	Type of cutter	Surface roughness R_a(μm)
n_0=270 r/min V_f=94 mm/min a_p=4 mm	Equal blade cutter	91
	Unequal blade cutter	43

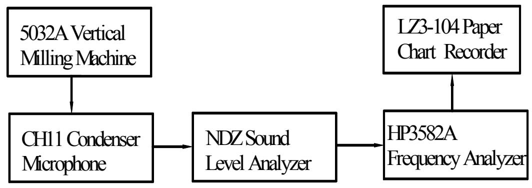

Fig. 7. Noise measurement set-up.

show that the noise level was reduced by 3dB using the irregular-pitch cutter.

Finally, the surface roughness of the workpiece was measured. As stated above, the material of the workpiece was #45 steel of HB187 hardness, a 300 mm × 160 mm × 100 mm block of which was cut on the milling machine. The cutting conditions were spindle speed n_0 = 270 r/min, feed speed v_f = 94 mm/min, and depth of cut a_p = 4 mm. Table 3 lists the experimental results, which show that the surface roughnesses (R_a) for the irregular- and regular-pitch cutters were 43 μm and 91 μm , respectively. These data are clear proof that the irregular-pitch cutter can enhance the surface-finish quality of a workpiece.

7. Conclusions

The optimum design of an irregular-pitch face-milling cutter for reduction of vibration force and noise was investigated. A cutting-force model and objective function for optimum blade spacing were set up. Based on a theoretical analysis of the milling force spectrum, the vibration-free mechanism of an irregular-pitch face-milling cutter was studied. A general-purpose face-milling cutter of uneven blade spacing was then designed, and its performance was compared with that of a regular-pitch cutter. The data showed that the irregular-pitch cutter effected not only a significant reduction in the relative vibration and acoustical noise level but also a remarkable improvement in the quality of the surface-finish of the workpiece.

Acknowledgments

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